N 69 14223

NASA TECHNICAL TRANSLATION NASA TT F-12,127

NASA TT F-12,127

Page One Title

DETERMINATION OF THE PARAMETERS OF THE SELF-OSCILLATION PROCESS IN A STEAM-GENERATING CONDUIT

K. N. Proskuryakov

CASE FILE COPY

Translation of "Opredeleniye Parametrov Avtokolebatel'nogo Protsessa v Parogeneriruyushchem Kanale" Izvestiya Vysshikh Uchebnykh Zavedeniy Energetika, No. 10, pp. 92-96, 1965

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 DECEMBER 1968

NASA TT F-12,127

DETERMINATION OF THE PARAMETERS OF THE SELF-OSCILLATION PROCESS IN A STEAM-GENERATING CONDUIT

K. N. Proskuryakov

ABSTRACT. The conditions of self-oscillation in a steam-generating system equipped with an economizer and vaporizer are investigated for the case two-phase flow. The relations employed for expressing these conditions are described. It has been established that the periodic parametric pulsations occurring in steam-generating conduits under certain conditions are self-oscillations, and that a high amplitude of pulsation in the heat flux on the steam-generating surface of the heat exchanges corresponds to a large magnitude of the parameter characterizing the damping of oscillations in a two-phase flow.

Parametric oscillations occurring during the operation of heat exchangers /92* during the generation of steam are of great technical interest at the present time, since the achievement of certain amplitudes of parametric change is often associated with limitations on operating conditions.

It has been established experimentally [1-3] that when a definite set of geometric conditions is imposed on steam-generating systems and operating conditions such as pressure, flow rate of the heat carrier, and heat flux on the heated surface, parametric oscillations occur in the two-phase flow which are close to harmonic. Periodic changes in the parameters characterizing the operation of a steam-generating conduit in the absence of a periodic external load indicate that the steam-generating conduit is a self-oscillating system in this case.

As a consequence of self-oscillations in a steam-generating conduit equipped with an economizer and vaporizer, oscillations occur in the boiling point at some average position (Figure 1). Up to the boiling point, the flow of the heat carrier is practically incompressible, but after the boiling point, the flow becomes two-phase and possesses higher compressibility. The effect of the steam-and-water volume on the oscillating mass of the heat carrier is similar to the action of a spring. With this in mind, we used the experimental model illustrated in Figure 2.

---

* Numbers in the margin indicate pagination in the foreign text.

Self-oscillations occur in a steam-generating conduit because of non-linearity in the two-phase flow. For this reason it is possible to establish oscillations of finite amplitude, since otherwise (in the absence of non-linearties) the amplitude of the initial exitations would be infinite, and this would lead to the destruction of the system.

The differential equation describing free oscillations of the mass without consideration of non-linear factors

$$m\ddot{y} + Cy = 0, \tag{1}$$

can be written with consideration of non-linear factors and damping in the system in the following form:

$$\ddot{y} + \varepsilon\omega_0\dot{y} + \omega_0^2 F(y) = p(t). \tag{2}$$

Here y is a dimensionless permutation, which can always be transformed into a dimension defined as the deviation of the boiling point from the state of equilibrium. In order to obtain the dimensionless permutation, we divide the magnitude of deviation from the state of equilibrium by the total length of the two-phase section; ω_0 is the frequency of natural oscillations of the /93
system ($\omega_0^2 = C/m$); ε is a parameter characterizing damping in the system, i.e. characterizing energy losses; $\dot{y}$ is the rate of permutation; $\ddot{y}$ is acceleration of permutation; p(t) is the external effect as some function of time; F(y) is a function considering the effect of non-linear factors.

A steam-generating conduit is an inertial system, and therefore there will be a time shift between the external effect on the two-phase flow in the form of a heat load on the heated surface and the reaction of the steam-and-water volume. It may turn out as the consequence of this shift that the external effect is supported by the system itself, i.e. the sequential intake of energy governs the deviation of the boiling point from the equilibrium position, and conversely. For example, a reduction in the steam-and-water volume, which is manifested as a displacement of the boiling point in the direction of the movement of the flow, in the final analysis alters the coefficient of heat transfer from the wall to the flow, and this in turn alters the amount of heat energy delivered to the flow. As a consequence of this change, the boiling point is shifted in the reverse direction. Thus a feedback develops between the amount of energy absorbed by the heat carrier from without and the reaction of the flow to this effect.

In our case this singularity, which is a feature of self-oscillating systems in general, can be expressed as follows:

$$\omega_0^2 y = p(t - \tau), \tag{3}$$

Page One Title
Cover Page Title

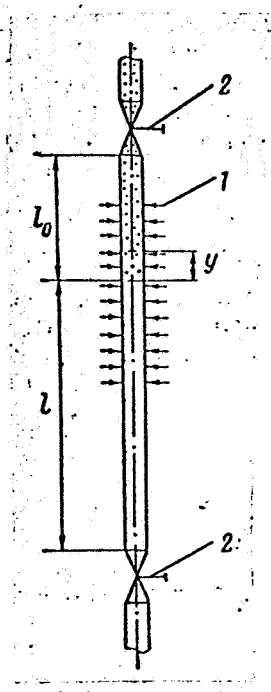

Figure 1. Schematic Diagram of System With Steam-Generating Conduit; 1--Heat Flux; 2--Valve.

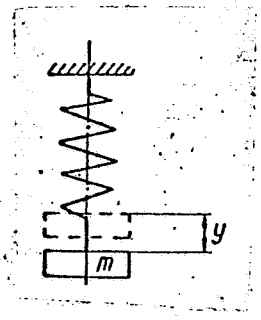

Figure 2. Schematic Model of System With Steam-Generating Conduit.

cluding self-oscillating systems.

For the non-linear function

where τ is the lag time. Condition (3) is an expression of the fact that the steam-generating conduit is a closed system, i.e. the action "at the intake" is equal to the action "at the outlet", and consequently the heat energy taken on by the flow, and corresponding to p(t), is uniquely related to the deviation of the boiling point y from the equilibrium position. We may assume on the basis of equation (3) that there exists a periodic solution for equation (2):

$$\begin{aligned} p'(t) &= Be^{i\omega t}; \\ \omega_0^2 y &= De^{i\omega t}. \end{aligned} \qquad (4)$$ /94

In order to find the solution of equation (2) in the form (4), it is necessary to substitute the non-linear term F(y) by some equivalent linear term:

$$F(y) = q(A)\,y. \qquad (5)$$

To determine the coefficient q(A), it is convenient to use the method of harmonic linearization, which is an effective and reliable method for analyzing non-linear dynamic systems in general, in-

$$F(y) = p_0\left[\frac{1}{(1-y)^k} - 1\right], \qquad (6)$$

which is the accurate non-linear characteristic of the model examined herein, we are completely justified in limiting ourselves to the first three terms of its expansion into the Taylor series [4] for practical calculations. Then, by the method of harmonic linearization for the coefficient q(A), we derive the formula

$$q(A) = k + 0{,}85\frac{k(k+1)}{2}A + 0{,}75\frac{k(k+1)(k+2)}{6}A^2 \ldots \tag{7}$$

Here k is the isoentropic index (adiabatic).

Equation (2) is now written in the linearized form as

$$\ddot{y} + \varepsilon\omega_0\dot{y} + q(A)\,\omega_0^2 y = p(t), \tag{8}$$

where q(A) is defined by formula (7). We will now find the periodic solution (4) for equation (8). Since on the basis of (4)

$$y = \frac{1}{\omega_0^2} De^{i\omega\tau}, \tag{9}$$

then by substituting its value in equation (8) we find

$$D\frac{(i\omega)^2}{\omega_0^2}e^{i\omega t} + D\varepsilon\frac{i\omega}{\omega_0}e^{i\omega t} + Dq(A)\,e^{i\omega t} = Be^{i\omega t}. \tag{10}$$

With consideration of (3)-(4)

$$D = Be^{-i\omega\tau}. \tag{11}$$

Then, instead of equation (10), we write

$$\frac{(i\omega)^2}{\omega_0^2} + \varepsilon\frac{i\omega}{\omega_0} + q(A) = e^{i\omega\tau}. \tag{12}$$

Equation (12) is the characteristic equation for our system with lag. In the usual linear systems without consideration of lag τ, the characteristic equations of the type (12) are ordinary algebraic equations. But in our system with lag, the characteristic equation is transcendental, due to the exponential function in the right-hand half of the equation. Equation (2) can be reduced by elementary transformations into the following relation:

$$\left(\varepsilon\frac{\omega}{\omega_0}\right)^2 + \left[q(A) - \frac{\omega^2}{\omega_0^2}\right]^2 = 1. \tag{13}$$

The case where self-oscillations are established in a conduit with a frequency equal to that of the natural oscillations involved in the oscillation of the mass of the heat carrier is of great practical interest. The condition for the existence of self-oscillations can then be written in the following form:

Cover Page Blank

$$[q(A) - 1]^2 + \varepsilon^2 = 1. \tag{14}$$

Relation (14) is quite graphic in the physical sense. It seems that the /95
amplitude of self-oscillations A depends on the form of non-linearity and on the parameter of damping ε, but is absolutely independent of all other parameters. In turn, the amount of energy required for maintaining self-oscillations with a given amplitude and frequency is uniquely determined by this amplitude and frequency. On the basis of equation (3), and in observance of (4), we have

$$\omega_0^2 A = Be^{-i\omega_0\tau}, \tag{15}$$

where B is the amplitude of the exterior action, which varies in a sinusoidal manner. By setting the real parts in equation (15) equal, we find

$$B\cos\omega_0\tau = \omega_0^2 A, \tag{16}$$

after which we can now write the expressions for the determination of the amplitude of the effect of the heat flux in explicit form:

$$B = \frac{\omega_0^2 A}{\cos\omega_0\tau}. \tag{17}$$

In the right half of equation (15) there still remains the undefined lag parameter τ. This can be found by equating the imaginary terms in the characteristic equation of self-oscillations (12):

$$i\varepsilon\frac{\omega}{\omega_0} = i\sin\omega\tau. \tag{18}$$

In our partial case, where $\omega = \omega_0$, the writing of formula (18) for the definition of τ is somewhat simplified:

$$\sin\omega_0\tau = \varepsilon. \tag{19}$$

By using the trigonometric function

$$\cos\omega_0\tau = \sqrt{1-\sin^2\omega_0\tau}, \tag{20}$$

then, on the basis of (16) and (19), we may write

$$B = \frac{\omega_0^2 A}{\sqrt{1-\varepsilon^2}}. \tag{21}$$

In view of the fact that the natural frequency of oscillations of the system can be expressed by the parameters of the steam-generating conduit in accordance with [5], as follows:

$$\omega_0 = \sqrt{\frac{kp_0l_0}{\rho lf}}, \tag{22}$$

then it is possible to derive the expression for the amplitude of variation of the heat flux on the surface of the conduit as a function of the specific dimensions and parameters of the system with a single steam-generating conduit:

$$B = \frac{Ak\,p_0l_0}{\rho l\,f\sqrt{1-\varepsilon^2}}, \tag{23}$$

where p_0 is the average pressure for the period of oscillations; l_0 is the length of the steam-and-water column, which plays the part of a spring; l is the length of the channel filled with liquid from the point of constant pressure to the beginning of the vaporizer; ρ is the density of the liquid; f is the unit of the cross-section of the channel.

From the relation thus derived it follows that in order to maintain the self-oscillating condition of the steam-generating system, the greater the coefficient of damping ε, the more energy must be expended. This is in complete agreement with the physical representations.

Conclusions

1. It has been shown that the periodic pulsations of parameters occurring /96 under certain conditions in steam-generating conduits are self-oscillations.

2. The conception of a dynamic system with lag and with a positive feedback has been used to describe the self-oscillating process in a steam-generating conduit.

3. The relations of the parameters at which stable self-oscillations are possible have been derived.

Page One Title

4. The relations for the calculation of the amplitude of the external action and lag time have been derived.

Header Page Title

5. It has been shown that a high amplitude of pulsations in the heat flux on the steam-generating surface of the heat exchanger corresponds to a large magnitude of the parameter characterizing damping of oscillations in a two-phase flow. The conclusion that the frequency of pulsations of the heat flow is equal to the frequency of self-oscillations, i.e. in the given case of the natural frequency involved in the oscillations of the mass of the heat carrier, is also obvious.

REFERENCES

1. Morozov, I. I. and V. A. Mal'kov, *Inzhenerno-fizicheskiy Zhurnal*, No. 1, 1963.
2. Styrikovich, M. A. and Z. L. Miropol'skiy, et. al., *Teploenergetika*, No. 5, 1960.
3. Meyer, I. E. and R. P. Rose, Paper Amer. Soc. Mech. Engrs., No. HT-41, p. 9, 1962.
4. Proskuryakov, K. N., *Teploenergetika*, No. 3, 1965.
5. Proskuryakov, K. N., *Trudy MEI*, No. 63, 1965.

Translated for the National Aeronautics and Space Administration under contract No. NASw-1695 by Techtran Corporation, P.O. Box 729, Glen Burnie, Maryland 21061